Hooke's Law: Nonlinear Generalization and Applications

Part III Distribution of Stress in Prisms and Beams

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Introduction

As stated in [5] Part II, the distribution of stress σ in the bending pressure zone of a beam is identical with the distribution of stress in a non-centric loaded prism in such a manner, that the strain ε_2 , remote to the loading, is zero (see figure 1).

In a huge number of experiments with the variables

$\varepsilon_1(^\circ/_{\circ\circ})$=$\varepsilon_0$ (in contrast to $\varepsilon_2 = 0$)

x=1-β_0, the center of loading; NB! The width of the prism d=150cm (see [5], Part II, figure 2) is standardized to 1: 0≤ x=≤1=d/150,

α_0^p=$\bar{\sigma}/K_b$, the (relative) loading ; K_b(kg/cm^2) is the utmost strength of the prism with centric loading - Generally: Index 0 means „experiment with ε_2=0",

in [2] the experimental results are given in dependence on strength of cube W=W^{20} (kg/cm^2; cube length 20 cm) and κ=$\bar{\sigma}/\bar{\sigma}_B$ (B for break). See also tables 1, 2, 3 and figures 4, 5 and 6 of [5], part II.

With these experimental data 3 hypothesis were postulated in [5], part II (hypothetical values are signed as $\hat{v}$ in contrast to experimental values v – as it is usus in statistics).

$$\textbf{Hypothesis H1:}\quad \hat{\beta}_0(W,\kappa) = 1/3 + (1/2-1/3)\, e^{-a(\frac{W}{100})}\, \kappa^b$$

$$\textbf{Hypothesis H2:}\quad \hat{\alpha}_0^P(W,\kappa) = (2/3)\kappa + (1/3)\, e^{-a(\frac{W}{100})}\, \kappa^b$$

$$\textbf{Hypothesis H3:}\quad \hat{\kappa}(\varepsilon_0) = a\, \varepsilon_0\, e^{-b\varepsilon_0}$$

The good coincidence of experiments and hypotheses can be seen in the tables and figures of [5], part II.

Distribution of stress σ *in an eccentric pressed prism (with* ε_2=0)

The (relative) stress y(x)=σ(x)/K_b is

$$\textbf{Hypothesis H4:}\quad y(x) = Axe^{-Bx}$$

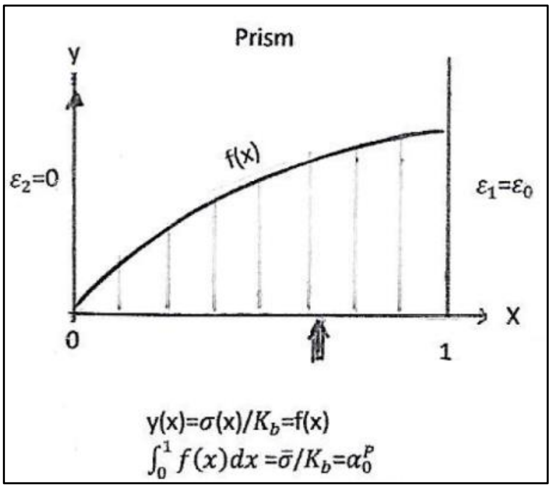

Fig. 1: Sketch of an eccentrically pressed prism with center of pressure x=1-β_o so, that ε_2 =0

see figure 1. That means: The stress-function $\sigma(x)$ for eccentric pressed prisms is of the same mathematical form as for centric pressed prisms, as K_b, as well as W, is a constant for a fixed experiment.

The two parameters A and B are estimated by minimizing according to the method of least squares of Gauss

$$D_1=(\int_0^1 y(x)dx - \hat{\alpha}_0^P)^2 \qquad \text{and}$$

$$D_2=\left[\left(\int_0^1 xy(x)\,dx\right) / \int_0^1 y(x)dx-(1-\hat{\beta}_0)\right]^2$$

together by minimizing the objective function D=D_1+D_2.

The values of $\hat{\alpha}_0^P$ and $\hat{\beta}_0$ are given in [5], part II, tables 2 and 1 for a series of values of W and of κ.

For example for W=300 and κ=0,9 we have 0.653 and 0.374.

The minimum of D is found by the iterative nonlinear Simplex-method of Nelder and Mead [1]. The resulting optimal parameters A and B of hypothesis H_4 are registerd together with figure 2 for W=80, figure 3 for W=160, figure 4 for W= 300, figure 5 for W=450 and figure 6 for W= 600 (kg/cm^2). To every value of W seven values of κ= $\frac{\bar{\sigma}}{\bar{\sigma}_B}$ = 0.3, 0.4, … 0.9 were chosen for stress-lines. For $\kappa > 0.9$ and W< 80 I do not put my hand in the fire with my formulae. This may already be seen in figure 5b of [5], Part II for $\kappa = 0.9$ *and small values of W*.

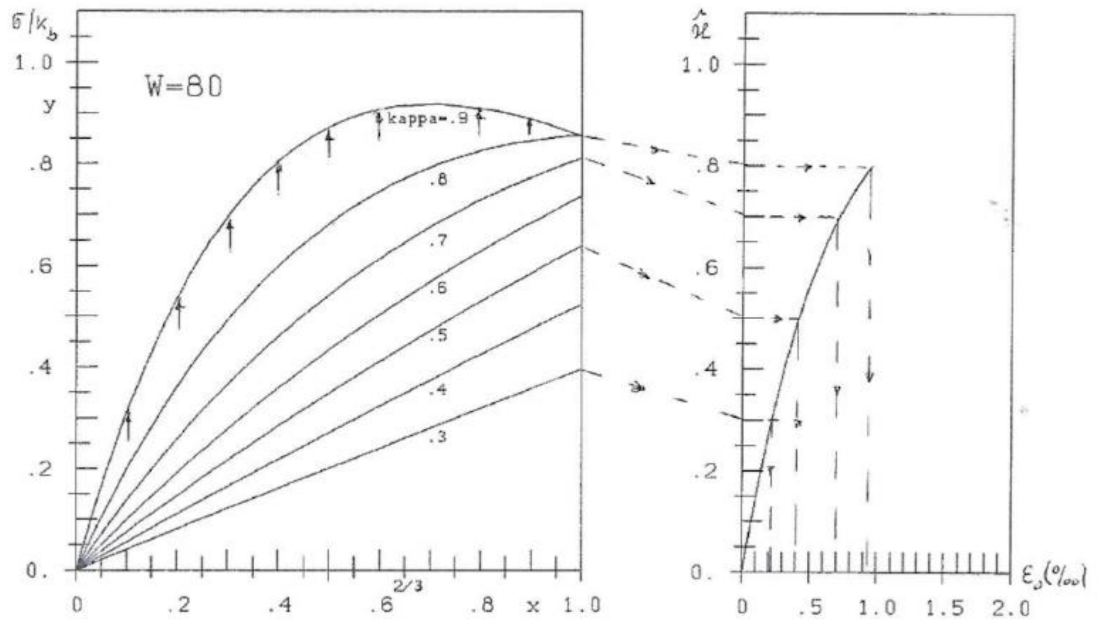

Fig. 2: W=80 kg/cm^2 . stress-lines $y = Axe^{-Bx}$ and strain ε_o according to H3 ([5], part II): $\hat{\kappa}$=a$\varepsilon_o e^{-b\varepsilon_o}$

κ	0.3	0 .4	0 .5	0 .6	0 .7	0.8	0.9
A	0.4082	0.5601	0.7577	1.0230	1.4540	2.1750	3.5630
B	0.0306	0.0663	0.1726	0.3295	0.5850	0.9321	1.4280

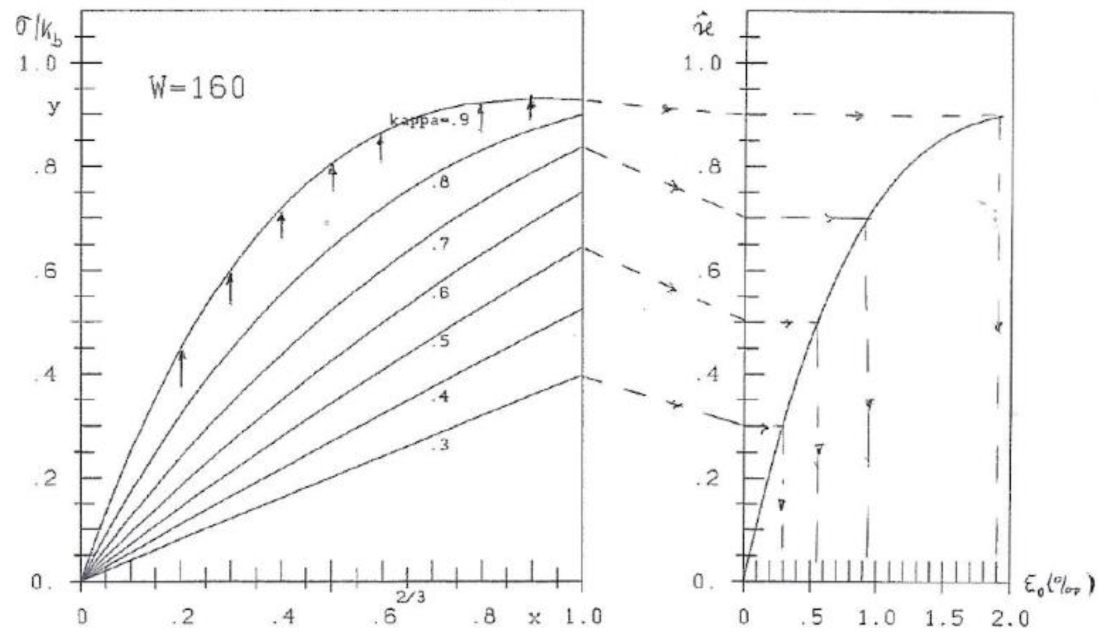

Fig. 3: W=160 kg/cm^2 . stress-lines $y = Axe^{-Bx}$ and strain ε_o according to H3 ([5], part II): $\hat{\kappa}$=a$\varepsilon_o e^{-b\varepsilon_o}$

κ	0.3	0 .4	0 .5	0 .6	0 .7	0.8	0.9
A	0.4057	0.5551	0.7325	0.9661	1.3080	1.8530	2.7990
B	0.0199	0.0556	0.1284	0.2532	0.4463	0.7235	1.1050

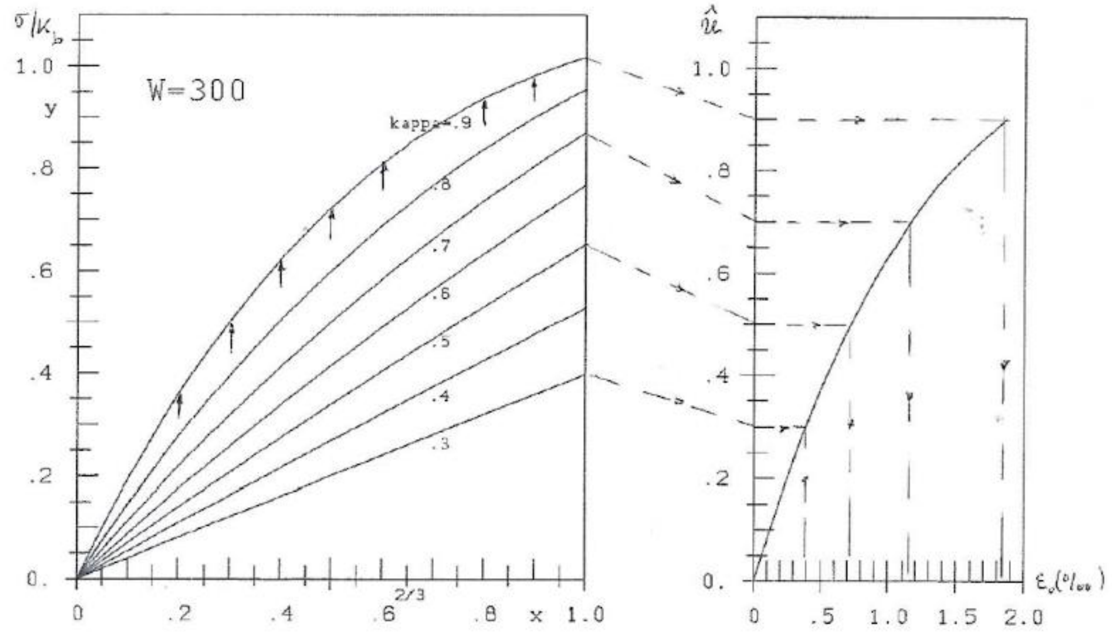

Fig. 4: W=300 kg/cm^2 . stress-lines $y = Axe^{-Bx}$ and strain ε_o according to H3 ([5], part II): $\hat{\kappa}$=a$\varepsilon_o e^{-b\varepsilon_o}$

κ	0.3	0 .4	0 .5	0 .6	0 .7	0.8	0.9
A	0.4035	0.5470	0.7070	0.8996	1.1530	1.5110	2.0590
B	0.0126	0.0358	0.0805	0.1586	0.2810	0.4581	0.7037

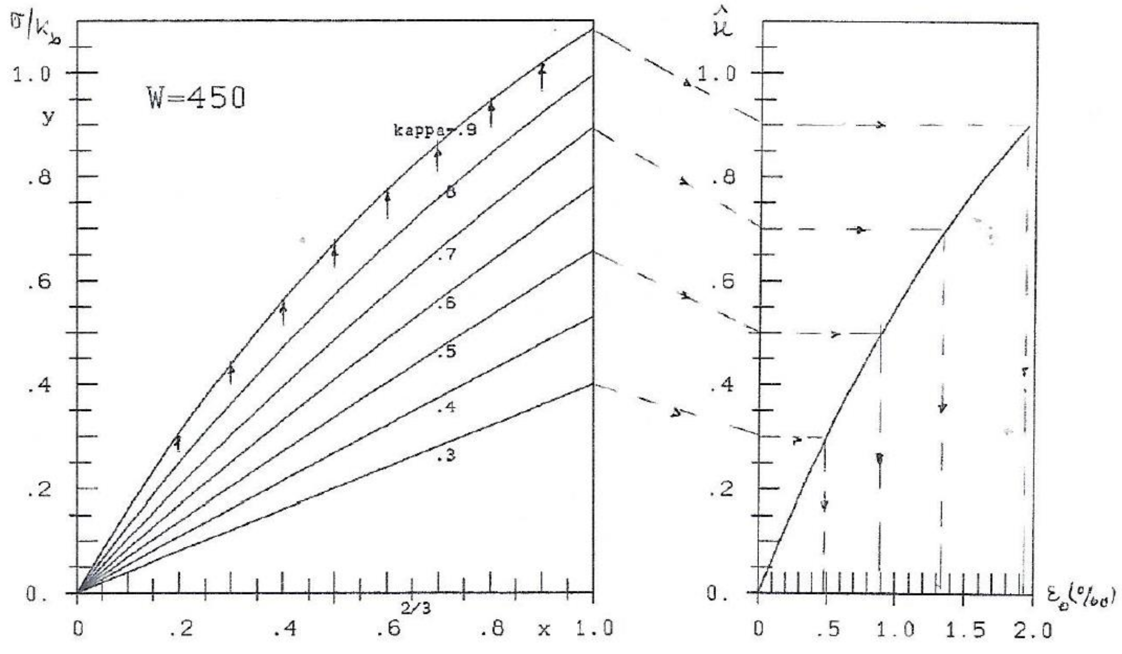

Fig. 5: W=450 kg/cm^2 . stress-lines $y = Axe^{-Bx}$ and strain ε_o according to H3 ([5], part II): $\hat{\kappa}$=a$\varepsilon_o e^{-b\varepsilon_o}$

κ	0.3	0 .4	0 .5	0 .6	0 .7	0.8	0.9
A	0.4019	0.5415	0.6904	0.8587	1.0600	1.3170	1.6680
B	0.0071	0.0217	0.0485	0.0966	0.1708	0.2792	0.4309

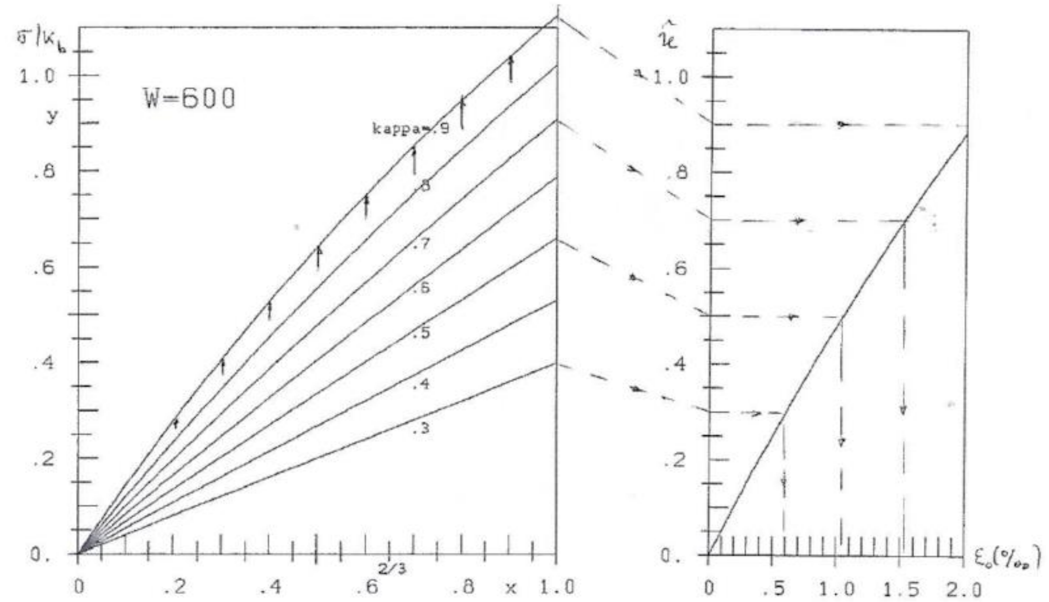

Fig. 6: W=600 kg/cm^2 . stress-lines $y = Axe^{-Bx}$ and strain ε_o according to H3 (([5], part II): $\hat{\kappa}$=a$\varepsilon_o e^{-b\varepsilon_o}$

κ	0.3	0 .4	0 .5	0 .6	0 .7	0.8	0.9
A	0.4014	0.5380	0.6813	0.8341	1.0080	1.2110	1.4650
B	0.0054	0.0125	0.0306	0.0573	0.1038	0.1691	0.2636

In the left part of the figures the disrtribution of stress y is given.

The strain ε is zero for x=0 according to the experiments. For x=1 strain ε=ε_0 can be computed with formulae $\hat{\kappa}$=a$\varepsilon_0 e^{-b\varepsilon_0}$ of [5], part II. This is shown in the right part of the figures. For example in the experiment with W=300 and some values of $\hat{\kappa}$ we get (see also figure 4):

$\hat{\boldsymbol{\kappa}}$	.3	.4	.5	.6	.7	.8	.9
$\boldsymbol{\varepsilon_0}$	.39	.55	.72	.92	1.16	1.45	1.86

Distribution of stress σ in the bending pressure zone of a beam

According to Part II the distribution of the stress in the bending pressure zone of a beam is identical with that of a corresponding eccentric pressed prism with ε_2=0: The side of the prism with ε=0 (y-axis in figure 1) is equivalent with the neutral axis of the beam (see figure 7).

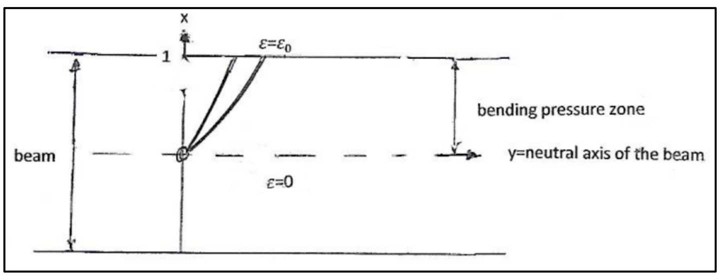

Fig. 7: Sketch of the bending pressure zone of a beam

In figures 8, …, 12 the lower part give the stress-lines in the bending pressure zones of the beams. They simply are the mirror-images of the stress lines of the corresponding prisms, reflected at the mirror with y=x.

The curves $\hat{\kappa}$=a$\varepsilon_0 e^{-b\varepsilon_0}$ in the upper part of the figures are the same as those of the corresponding prisms (in a somewhat modified scale).

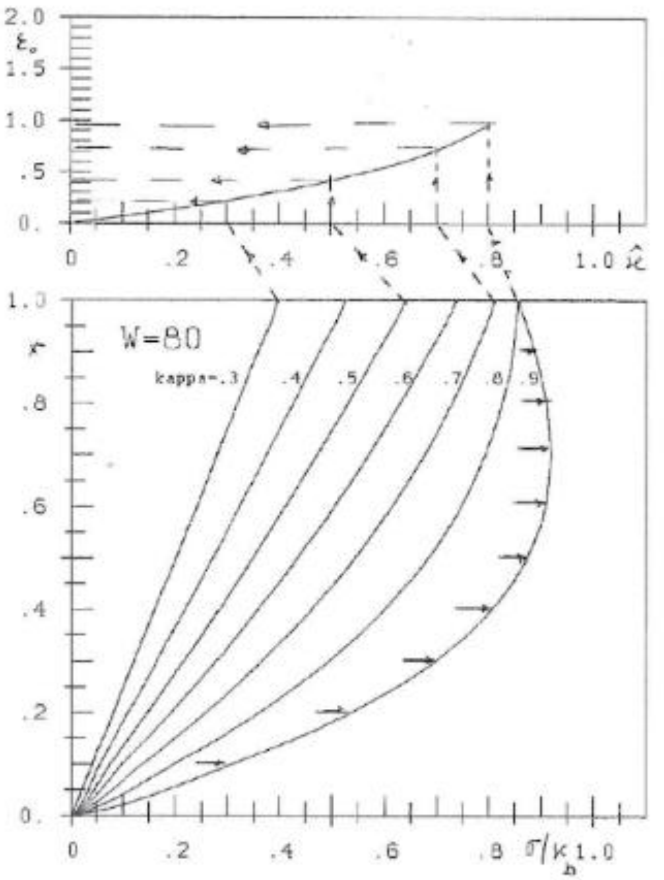

Fig. 8: W=80 kg/cm^2. Strain ε_o and stress-lines σ/K_b
In the bending pressure zone $0 \le x \le 1$

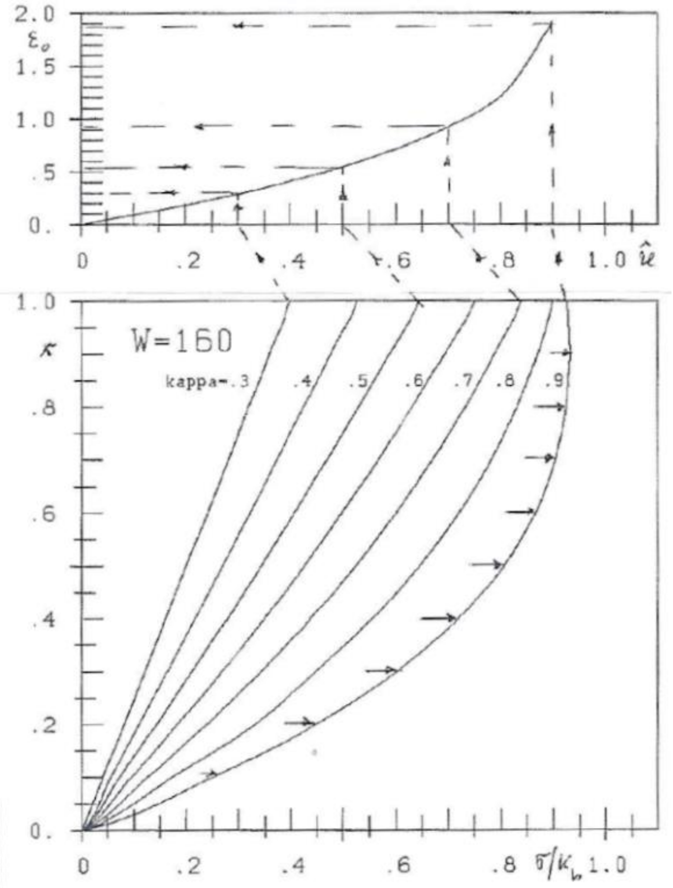

Fig. 9: W=160 kg/cm^2. Strain ε_o and stress-lines σ/K_b
In the bending pressure zone $0 \le x \le 1$

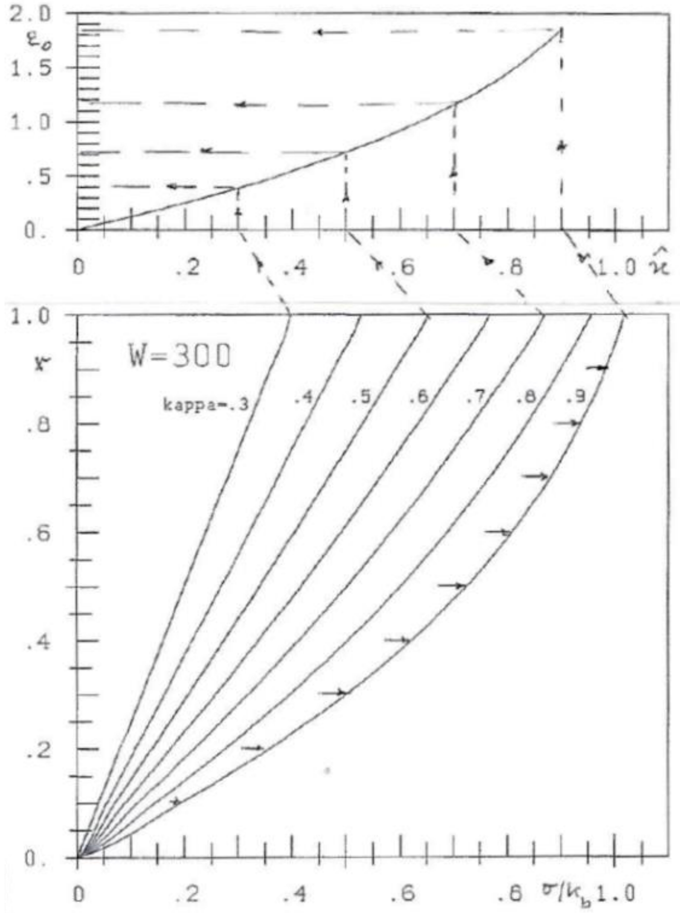

Fig 10: W=300 kg/cm^2. Strain ε_o and stress-lines σ/K_b
In the bending pressure zone 0$\leq x \leq 1$

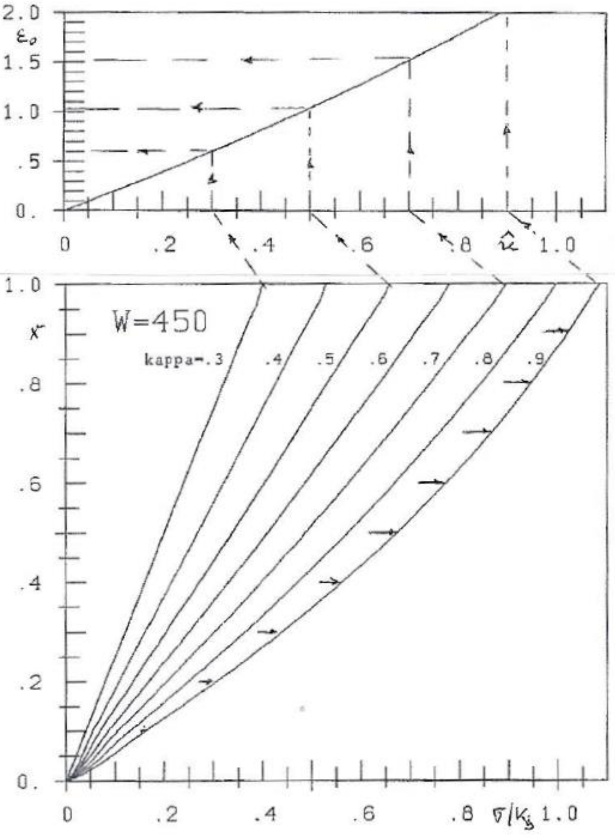

Fig. 11: W=450 kg/cm^2. Strain ε_o and stress-lines σ/K_b
In the bending pressure zone 0$\leq x \leq 1$

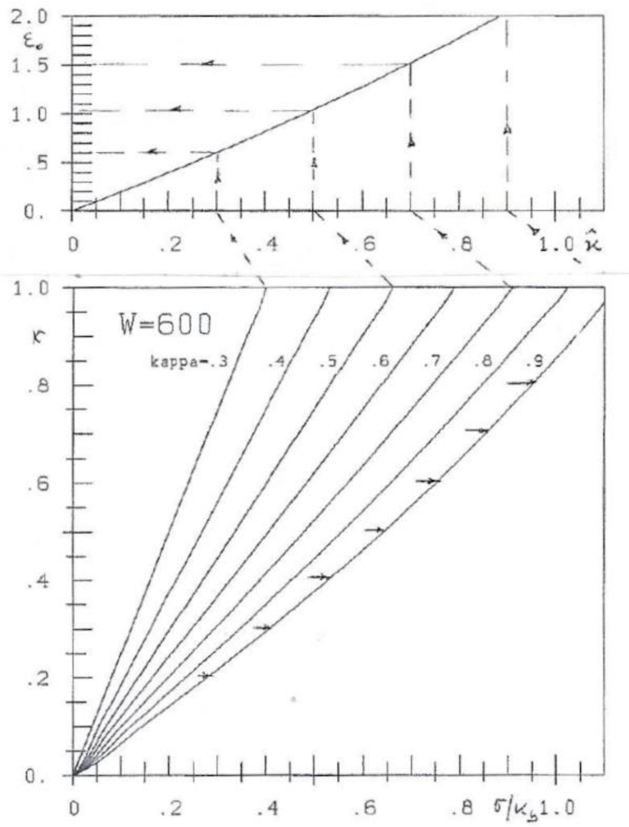

Fig. 12: W=600 kg/cm^2. Strain ε_o and stress-lines σ/K_b
In the bending pressure zone 0$\leq x \leq 1$

Comment

The experiments, the results of which are used in this analysis, were directed by Rüsch [2] at the University of Technology Munich. The evaluation of the data was done under the direction of Scholz, who also made two hypotheses on the distribution of the stress in beams ([3] and [4]).

References

[1] Nelder J.R. and Mead R. (1965). A Simplex-Method for Function Minimization. The Computer Journal 7, 303-313

[2] Rüsch H. (1955). Versuche zur Festigkeit der Biegedruckzone. Deutscher Ausschuss für Stahlbeton, Heft 120

[3] Scholz G.(1957). Über die wahrscheinlichste Spannungsverteilung in der Biegedruckzone von Stahlbetonbalken mit rechteckigem Querschnitt. Dissertation

[4] Scholz G. 1960). Festigkeit der Biegedruckzone, Heft 139. Deutscher Ausschuss für Stahlbeton, Heft 139

[5] Schneeberger H.(2020). Hooke's Law. Nonlinear Generalization and Applications. https://www.hookes-law-generalized.de